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THERMODIFFUSIOPHORETIC CAPTURE OF AEROSOL PARTICLES IN A PLANE CHANNEL WITH NONUNIFORM TEMPERATURE

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The article theoretically studies the process of capturing aerosol particles from a laminar stream of a binary gas mixture inhomogeneous in temperature and concentration passing through a plane channel with nonuniform temperature.

Aerosol particles are settling in a channel through which passes a stream of a binary gas mixture; its first component consists of molecules of some volatile substance condensing on the lower plate with a temperature T_h that is lower than the temperature of the upper plate T_0. We examine the case of steady-state motion of the gas stream where we may neglect the influence of the inlet part on the distribution of mass velocity, temperature, and concentration of the components of the gas mixture. The theory of capture is devised for gas mixtures with similar molecular masses where the coefficients of viscosity, thermal conductivity, and diffusion depend only weakly on the concentration of the substances of which the gas mixture is composed. Among such gas mixtures is the steam–air mixture consisting of molecules of air and water vapor.

The aerosol particles entering the channel begin to move toward the surface of the lower plate along a path described by the differential equation of motion of aerosol particles

$$dx/v_x = dz/v_z, \tag{1}$$

where v_x and v_z are the x- and z-components of the velocity of the particles. The velocity of steady-state motion of the particles relative to the channel walls is composed of the velocity of mass motion of the gas **u**, the speed of diffusiophoresis due to nonuniform distribution of the concentration $\mathbf{v}_D$ [1, 2], the velocity $\mathbf{v}_T$ due to nonuniform distribution of the temperature T [1, 2], and the gravitational velocity $\mathbf{v}_g$:

$$\mathbf{v} = \mathbf{u} + \mathbf{v}_D + \mathbf{v}_T + \mathbf{v}_g = \mathbf{u} - \frac{D_{12} f_D}{c_2} \operatorname{grad} c_1 - f_T \frac{\nu}{T} \operatorname{grad} T - \frac{2}{9} f_g g \rho_i \frac{R^2}{\rho\nu} \mathbf{n}_x, \tag{2}$$

where $c_1 = n_1/n$; $c_2 = n_2/n$; $n = n_1 + n_2$; n_1 and n_2 are the concentrations of molecules of the first and second kind, respectively; m_1 and m_2, molecular masses of the first and second component, respectively; ρ_i, density of the substance of the particle; $\mathbf{n}_x$, unit vector. The scalar coefficients f_D, f_T, and f_g depend on the geometrical dimensions of the particles, on the phase composition of the substance of the particles, on c_1, c_2, T, and on the molecular properties of the gas mixture. We do not present here the explicit form of the expressions for the coefficients f_D, f_T, and f_g because in the general case these expressions have a fairly cumbersome form. The expressions for f_D, f_T, and f_g can be found in [1-4]. In the model of gas flow examined here, the distributions of u_x, u_z, T, and c_1 depend only on the x-coordinate. Therefore $v_z = u_z$, and v_x is described by the expression

$$v_x = u_x + v_{Dx} + v_{Tx} + v_{gx} = u_x + f_D D_{12} \frac{d}{dx} \ln(1 - c_1) - \nu f_T \frac{d}{dx} \ln T - \frac{2}{9} g f_g \rho_i \frac{R^2}{\rho\nu}. \tag{3}$$

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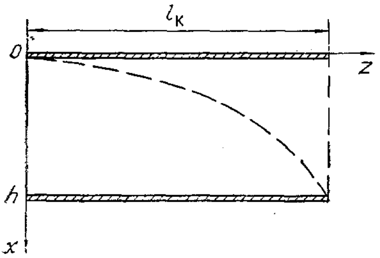

Fig. 1. Schematic diagram of the plane channel.

The path of the aerosol particles is bounded above by the limit path (see the curve in Fig. 1), and the length of its projection onto the horizontal plane determines l_K at which complete capture of the particles in the channel occurs. This path can be found when the expressions for u, T, and c_1 are known.

The distribution u in the channel, the pressures p, and c_1 and T in the stream under examination are described by the system of equations (4)-(6) which contains the system of hydrodynamic equations (4) and the equations describing mass transfer (5) and heat transfer (6) in the channel:

$$\frac{d}{dx}\,\rho u_x = 0;\quad \frac{\rho}{2}\,\frac{d}{dx}\,u_x^2 = -\frac{\partial p}{\partial x} + \frac{4}{3}\,\frac{d}{dx}\left(\mu\,\frac{du_x}{dx}\right), \tag{4}$$

$$\rho u_x\,\frac{du_z}{dx} = -\frac{\partial p}{\partial z} + \frac{d}{dx}\left(\mu\,\frac{du_z}{dx}\right),$$

$$\frac{d}{dx}\left(n_1 u_x - D_{12}\,\frac{n^2 m_2}{\rho}\,\frac{dc_1}{dx}\right) = -j, \tag{5}$$

$$\frac{d}{dx}\left\{kT\left[C_{p1}\left(n_1 u_x - D_{12}\,\frac{n^2 m_2}{\rho}\,\frac{dc_1}{dx}\right) + C_{p2}\left(n_2 u_x - D_{12}\,\frac{n^2 m_1}{\rho}\,\frac{dc_2}{dx}\right)\right] - \varkappa\,\frac{dT}{dx}\right\} = Lm_1 j, \tag{6}$$

where $\rho = m_1 n_1 + m_2 n_2$ is the density of the gas mixture; C_{p1} and C_{p2} are the heat capacities per one molecule of the first or second kind, respectively, with constant pressure;

$$j(x,\ z) = 4\pi \int_0^\infty R^2 \varphi(R,\ T,\ c_1)\, f(R,\ x,\ z)\, dR. \tag{7}$$

In (7), the function φ (R, T, c_1) describes the density of the stream of vapor molecules condensing (evaporating) on the surface of a drop. The function f describes the distribution over the dimensions of the aerosol particles.

The system (4)-(6) is integrated in squares only in two boundary cases: when the particles have a negligible effect on the distribution of c_1 and T (i.e., with j = 0) and in the case of substantial effect exerted by the particles on the distribution of c_1 and T, when the concentration of the vapors n_1 of the volatile component at each point of the gas stream may be considered equal to the concentration of saturated vapors n_{1S} (i.e., n_1 is a function uniquely depending on the temperature T). With j = 0, integration of (4)-(6) yielded the following expressions for the distributions of u_x, u_z, p, c_1, and T in the channel:

$$u_x = m_1 n D_{12} s/(\rho h), \tag{8}$$

$$u_z = \frac{\alpha h^2}{\mu\omega}\left[t - \frac{1-\exp(\omega t)}{1-\exp\omega}\right], \tag{9}$$

$$p = p_0 - \alpha z - \frac{\mu^2\omega^2}{h^2\rho}\left\{1 - \frac{\rho}{\rho_h} + \frac{4}{3}\,\frac{\mu}{D_{12}}\,\frac{(m_2 - m_1)}{\rho m_1}\left[(1-c_{10})\exp(st) - \frac{\mu_h}{\mu}\left(\frac{\rho}{\rho_h}\right)^2 (1-c_{1h})\right]\right\}, \tag{10}$$

$$c_1 = 1 - (1-c_{10})\exp(st), \tag{11}$$

$$T = (T_h - T_0)\,t + T_0, \tag{12}$$

where

$$\alpha = \mu\omega Q_2/(h^3 b n_{20}\psi);\quad n_{20} = (1-c_{10})\,n;\quad \psi = \frac{1}{s}\left(1 - \frac{1}{s}\right)\exp s - \frac{\exp s - 1}{s(1-\exp\omega)} + \frac{1}{s^2} + \frac{\exp(\omega+s) - 1}{(\omega+s)(1-\exp\omega)}. \tag{13}$$

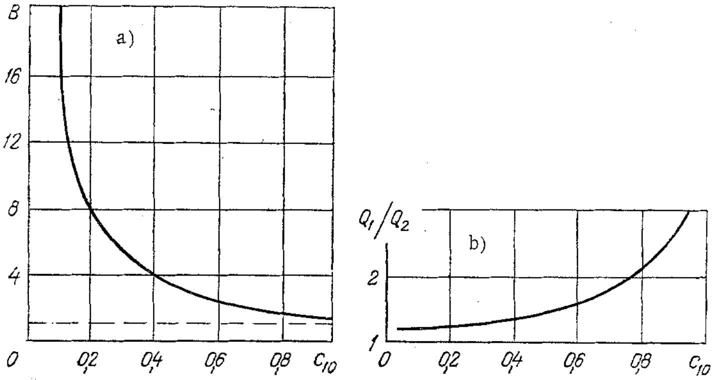

Fig. 2. Dependences of $B = l_K bnD_{12}(1+A)/(Q_2h)$ (a) and Q_1/Q_2 (b) on c_{10} for vapor–air mixture with $c_{1h} = 0.02$.

In (8)-(13), c_{10}, c_{1h} and T_0, T_h are the respective values of c_1 and T at the surfaces of the upper (c_{10}, T_0) and lower (c_{1h}, T_h) plates; $t = x/h$; $p_0 = p|_{z=0;\ t=1}$;

$$\omega = nD_{12}m_1 s/\mu;\ s = \ln\frac{(1-c_{1h})}{(1-c_{10})};\ \mu_h = \mu|_{t=1};\ \rho_h = \rho|_{t=1}.$$

When there is no stream of vapor ($c_{1h} \to c_{10}$) the value of the parameter s tends to zero. In that case $u_x = 0$, and (9) turns into Poiseuille's formula [6]:

$$v_z = \frac{\alpha_0 h^2}{2\mu} t(1-t), \tag{14}$$

where $\alpha_0 = 12\mu Q/(h^3 bn)$.

Expressions (8)-(13) were obtained on the assumption that $(T - T_h)/T \ll 1$ and that the distributions of c_1 and T along the surfaces of the plates are uniform. The smallness of the gradients permitted us to consider the transfer coefficients $\varkappa$, D_{12}, and μ to be independent of T.

When the aerosol particles have a substantial effect on the distribution of c_1 and T (i.e., the concentration of n in the stream may be considered equal to $n_{1S}(T)$), the values of c_1 and T in the channel can be found with the aid of the transcendental relationship obtained by joint integration of (5) and (6):

$$D_{12}\left(\tilde{T} + \frac{Lm_1}{C_{p1}k}\right)\left[\ln\frac{1-c_{1s}(T)}{1-c_{10}} - t\ln\frac{1-c_{1h}}{1-c_{10}}\right] + \theta[(T_0 - T) - t(T_0 - T_h)] = 0, \tag{15}$$

where $\tilde{T} = (T_0 + T_h)/2$; $c_{1S} = n_{1S}(T)/n$; $n_{1S}(T)$ is the concentration of saturated vapors at the temperature T; $\theta = \varkappa/(C_{p1}kn)$. The difference between the values c_1 and $c_{1S}(T)$ with specified t in a channel with constant c_{10}, c_{1h}, T_0, and T_h can be evaluated to advantage with the aid of the parameter

$$\varepsilon = \frac{|c_1 - c_{1s}(T)|}{c_1}. \tag{16}$$

In (16), the values of c_1 at the point with the coordinate t, with specified c_{10} and c_{1h}, are calculated by formula (11), and $c_{1S}(T)$ at the point t, with the same c_{10}, c_{1h}, T_0, and T_h, are found in the process of solving the transcendental equation (15). If ε, with $0 < t < 1$, is in absolute value much smaller than unity, the values of c_1 can be calculated by formula (11). As a rule, the maximum value of the parameter ε decreases with increasing T of the gas mixture. Here it should be pointed out that if an air–vapor mixture is indispensable for practical applications, formula (11) makes it possible to find the actual values of c_1 within a fairly broad interval of temperature changes. For instance, evaluations showed that in the case of channels with $c_{10} = c_{1S}(T_0)$, $c_{1h} = c_{1S}(T_h)$ for $T \geqslant 10°C$, the maximum error that may be permitted in finding the values of c_1 of water vapors with the aid of (11) cannot exceed 10%; for $T \geqslant 20°C$, 7%; and for $T \geqslant 30°C$, 5%.

The motion of the aerosol particles toward the surface of one of the plates is described by the differential equation (1). When the v_x- and v_z-components of v may be considered dependent only on the x coordinate

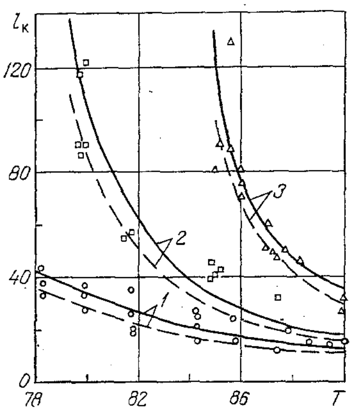

Fig. 3. Comparison of the curves of the dependence of l_K on T_0 with the experimental data of [5].

(this possibility is realized, e.g., when the distributions of c_1 and T are described by formulas (11) and (12) or by the relationship (15) and by the dependence of the coefficients f_D, f_T, and when the dependence of v_g on R may also be neglected), the length l_K may be found by the formula

$$l_к = \int_0^h \frac{v_z}{v_x}\, dx. \tag{17}$$

The integral on the right-hand side of (17) is expressed through elementary functions when the following conditions are fulfilled: a) the distributions u_x, u_z, c_1, and T are described by formulas (8), (9), (11), and (12), respectively; b) the variables f_D, f_T, ν, and v_{gx} change only slightly in the channel and may be considered constant; c) the value of the parameter ξ, determined by (18), may be considered smaller than unity ($\xi < 1$):

$$\xi = [(m_2 - m_1)\, Ac_2]/[(1 + A)\, m_1] < 1, \tag{18}$$

where

$$A = f_D + f_T \nu \frac{T_0 - T_h}{D_{12} s \tilde{T}} + \frac{v_{gx} h}{D_{12} s}; \ \tilde{T} = \frac{T_0 + T_h}{2}.$$

In gas mixtures with similar molecular masses (e.g., vapor–air mixture), when $|v_{gx}| < |v_{Tx} + v_{Dx}|$, the value of ξ is much smaller than unity. This makes it possible in the integration of (17) to confine oneself to the terms of zero and first orders of smallness with respect to the parameter ξ. As a result we obtain for l_K

$$l_к = l_{к0} \left\{ \left(\frac{1}{2} - \frac{1}{1 - \exp \omega} - \frac{1}{\omega} \right) \frac{1}{\psi c_{20}} + \frac{(m_2 - m_1)}{m_1} - \right.$$
$$\left. - \beta \left[1 + \frac{(m_2 - m_1)\, c_{2h}}{2\, m_1 s \psi \exp s} \left(\frac{\omega\, [1 - \exp(\omega + 2\, s)]}{(\omega + 2\, s)\, (1 - \exp \omega)} + \frac{1 - \exp(2\, s)}{2\, s} \right) \right] \right\} \equiv l_{к0} \Phi_1(s). \tag{19}$$

Here $l_{K0} = Q_2 h/[bnD_{12} s(1 + A)]$; $s = \ln(c_{2h}/c_{20})$; $\beta = [(m_2 - m_1)A]/[m_1(1 + A)]$, $\omega = nm_1 D_{12} s/\mu$.

Formula (19) assumes a simpler form when $\omega = s$:

$$l_к = l_{к0} \left\{ \frac{s}{c_{10} - c_{1h}} + \frac{m_2 - m_1}{m_1} - \beta \left[1 + \frac{c_{2h}(m_2 - m_1)}{m_1} F(s) \right] \right\} \equiv l_{к0} \Phi_2(s), \tag{20}$$

where

$$F(s) = \frac{[(1 + \exp s + \exp 2\, s)/3 + (1 - \exp 2\, s)/(2\, s)]}{2\, [1/2 + 1/s + (1/2 - 1/s) \exp s] \exp s}.$$

Expression (20) may be used, e.g., in calculations of l_K of channels with vapor–air mixture. When $\omega \geq s$, the function of l_K decreases monotonically with increasing c_{10} and T_0, tending to the limit

$$\lim_{s \to \infty} l_к = Q_2 h\, [1 + \mu/D_{12} m_1 n]/[2\, bnD_{12} c_{2h} (1 + A)]. \tag{21}$$

If in (21) we put c_{2h} equal to unity, we obtain the length l_K^* to which applies that l_K cannot be smaller than it. The nature of the decrease of l_K can be seen in Fig. 2a, where according to formula (20) the curve of the dependence of the function $B = l_K bnD_{12}(1 + A)/Q_2h$ on c_{10} for particles with $f_D = 0.2$, suspended in a stream of vapor–air mixture with $c_{1h} = 0.02$, was plotted. Here, the v_{Tx}- and v_{gx}-components of the velocity were disregarded because they are small [5].

When $\omega < s$, the function l_K with increasing c_{10} at first monotonically decreases, and then, when c_{10} has attained a certain value, it begins to increase monotonically (i.e., the function l_K for $\omega < s$ has a minimum). It can be seen from formulas (19) and (20) that when the values of h, b, c_{10}, c_{1h}, T_0, T_h are constant, the length l_K depends linearly on the throughput of scrubbed gas Q_2. When Q_2, c_{10}, T_0, c_{1h}, and T_h are constant, the length l_K depends linearly on h and changes inversely proportionally to b. The molecular stream of the volatile substance Q_1 (throughput), required for scrubbing the stream Q_2 in the case described by expressions (19) and (20), can be evaluated by the formula

$$Q_1 = l_K bnD_{12}s/h = Q_2\Phi_i(s)/(1 + A), \tag{22}$$

from which it can be seen that the stream Q_1 depends linearly on Q_2 and does not depend on the geometrical dimensions of the channel. An analysis of (22) showed that with increasing c_{10} (with constant values of Q_2, c_{1h}, T_0, and T_h) the function Q_1 increases monotonically (i.e., the minimum amount of the volatile substance is expended on scrubbing the polluted gas when c_{10} differs little in magnitude from c_{1h}). This is clearly shown in Fig. 2b which contains the curve of the dependence of the variable Q_1/Q_2 on c_{10} (22) for aerosol particles with $f_D = 0.2$ suspended in a vapor–air mixture with $c_{1h} = 0.02$. In calculating the values of Q_1/Q_2, the function $\Phi_2(s)$ was used. The minimum flow Q_1 of water vapor, required for scrubbing Q_2, can be found by a formula that was obtained by the limit transition $s \to 0$:

$$Q_{1_{\min}} = Q_2\left[\frac{1}{c_{2h}} + \frac{m_2 - m_1}{m_1}\right] \Big/ [(1 + A)(1 + \beta c_{2h})]. \tag{23}$$

The smallest value $Q_{1\min}$ is attained with $c_{2h} = 1$, and it is equal to $Q_2m_2/(m_1 + Am_2)$. It follows from (23) that when the process of scrubbing air can be described by the relationships (19) and (20), complete scrubbing of the air stream Q_2 requires a larger stream Q_1 of water vapor molecules (with $f_D = 0.2$; $Q_{1\min} = 1.25$).

Formulas (19) and (20) describe the capture of aerosol particles from gas mixtures unsaturated by vapors of the volatile substance (i.e., when the concentration of vapors of the volatile component at the surface of the upper plate is lower than the concentration of the saturated vapors corresponding to the temperature of the upper plate). Such conditions of scribbing can be created, e.g., by evaporating the molecules of the volatile component from the surface of the upper plate made of porous material, or by making the vapors in the channel pass through the pores of this plate.

When l_K is calculated by formulas (19) and (20), the mean values of the coefficients D_{12} and μ and of the concentration n have to be substituted in these formulas. These relationships may be used for calculating l_K and Q_1 when volatile drops or particles are captured in the channel (passing with $\varepsilon \ll 1$), if one of the following conditions is fulfilled:

$$v_x \gg v_{Dx} + v_{Tx} + v_{gx}, \tag{24}$$

$$v_x + v_{Dx} \gg v_{Tx} + v_{gx}, \quad v_{Dx} = \text{const}, \tag{25}$$

$$v_x + v_{Dx} + v_{gx} \gg v_{Tx}, \quad v_{Dx} = \text{const}, \quad v_{gx} = \text{const}. \tag{26}$$

When formulas (19) and (20) cannot be used for evaluating l_K, the boundary path of the particles can be found by numerically integrating the systems of equations (1), (4)-(6) to which the equation of the growth of drops has to be added:

$$\frac{4}{3}\pi\rho_i\frac{dR^3}{dt} = -4\pi R^2\varphi(R, T, c_1)m_1. \tag{27}$$

It should be pointed out that with the aid of (20) we can also approximately evaluate l_K when the drops grow in a condenser, where $\varepsilon \ll 1$. In that case we must substitute into (20) the minimum values of f_D, ν, f_T, and v_{gx}.

The experimental study of the settling of particles of tobacco smoke from a vapor–air stream in a plane-parallel channel was carried out in [5]. In the installation described there water vapor evaporated from the surface of the upper plate and condensed on the lower plate. The authors of [5] obtained satisfactory agreement between the experimental dependences of l_K on T_0 and Q_2 and the results of numerical calculations on a computer.

The distributions of c_1, T, and u in [5] were found by numerical integration of the system (4)-(6) which in the present work was integrated in quadratures. The authors of [5] took into account the dependence of μ, D_{12}, and $\varkappa$ on T. A comparison of the values of l_K, obtained in [5] by numerical integration, with the values of l_K calculated by formula (19) showed that they coincide well. The maximum difference between the values of l_K calculated by formula (19) and those taken from [5] (caused by the fact that in deriving (19) the dependence of the transfer coefficients on T was not taken into account) did not exceed 8%.

Figure 3 represents a comparison of the curves of the dependence of l_K on T plotted according to formula (19) with the experimental data presented in Figs. 3 and 12 in [5]. The experimental results were obtained with h = 2 cm, width of the plates b = 30.5 cm, mass throughput of air $Q_2^* = 0.0216$ g/sec for particles with a radius of $8 \cdot 10^{-5}$ cm. Curves 1 were plotted for $T_h = 68°C$, curves 2 for $T_h = 77°C$, curves 3 for $T_h = 83°C$. The solid lines in Fig. 3 were plotted without regard to the thermodiffusiophoretic motion of the aerosol particles (model 1 in [5]); the dashed lines with taking thermodiffusiophoretic motion of the aerosol particles into account (A = 0.2). It can be seen from Fig. 3 that the solid lines as well as the dashed lines agree within the experimental accuracy with the experimental data. The experimental values for $T_h = 68°C$ are denoted by rings, for $T_h = 77°C$ by rectangles, and for $T_h = 83°C$ by triangles.

NOTATION

T, temperature; x, z, Cartesian coordinates; t, dimensionless coordinate (t = x/h); v, particle velocity; u, mass velocity of the gas; c, relative concentration; n, concentration; ρ, density of the gas mixture; ρ_i, density of the substance; m, molecular mass; μ, coefficient of dynamic viscosity; ν, coefficient of kinematic viscosity; D_{12}, diffusion coefficient; $\varkappa$, thermal conductivity; R, radius of aerosol particle; g, acceleration of gravity; C_p, heat capacity with constant pressure; k, Boltzmann constant; L, temperature of phase transition; j, density of vapor effluence; Q, gas flow; p, pressure; h, distance between the plates forming the channel; b, width of the plates; l_K, length of the channel.

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